Don’t Split, Try To Work It Out : Bypassing Conflicts in Multi-Agent Pathfinding

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Abstract

Conflict-Based Search (CBS) is a recently introduced algorithm for Multi-Agent Path Finding (MAPF) whose runtime is exponential in the number of conflicts found between the agents’ paths. We present an improved version of CBS that bypasses conflicts thereby reducing the CBS search tree. Experimental results show that this improvement reduces the runtime by an order of magnitude in many cases.

Introduction and Overview

A Multi-Agent Path Finding (MAPF) problem is defined by a graph, \( G = (V, E) \) and a set of \( k \) agents labeled \( a_1, \ldots, a_k \), where each agent \( a_i \) has a start position \( s_i \in V \) and goal position \( g_i \in V \). At each time step an agent can either move to an adjacent location or wait in its current location. The task is to plan a sequence of move/wait actions for each agent \( a_i \), moving it from \( s_i \) to \( g_i \) such that agents do not conflict, i.e., occupy the same location at the same time. MAPF has practical applications in video games, traffic control, robotics etc. (See (Sharon et al. 2013) for a survey).

In this paper we focus on solving MAPF problems optimally, i.e., where the cost of the resulting plan is minimal. There is a range of algorithms that optimally solve different variants of MAPF using various search techniques (Standlee 2010; Wagner and Choset 2011; Sharon et al. 2013) or by compiling it to other known NP-complete problems (Surynek 2012; Yu and LaValle 2013; Erdem et al. 2013). Each of these solvers has pros and cons. There is no universal winner. Which algorithm performs best under what circumstances is an open research question.

Conflict-Based Search (CBS) (Sharon et al. 2012a), is an optimal MAPF solver shown to be very effective in many domains. It is a two-level algorithm. The low-level finds optimal paths for the individual agents. If the paths include conflicts, the high level, via a split action (described below), imposes constraints on the conflicting agents to avoid these conflicts. CBS is exponential in the number of conflicts seen.

This paper introduces an improved version of CBS. When a conflict is found, we first attempt to bypass the conflict and avoid the need to perform a split and add new constraints. If no bypass is found we resort to the drastic split action of adding explicit constraints to avoid the conflict. We provide a number of variants that search for bypasses differing in their search effort. Experimental results show speedups over basic CBS by more than an order of magnitude.

The Conflict Based Search Algorithm (CBS)

A sequence of individual agent move/wait actions leading an agent from \( s_i \) to \( g_i \) is referred to as a path, and the term solution refers to a set of \( k \) paths, one for each agent. A conflict between two paths is a tuple \( \langle a_i, a_j, v, t \rangle \) where agent \( a_i \) and agent \( a_j \) are planned to occupy vertex \( v \) at time step \( t \). A solution is valid if it is conflict-free. The cost of a path is the number of actions in it (including wait), and the cost of a solution is the sum of the costs of its constituent paths.

In CBS, agents are associated with constraints. A constraint for agent \( a_i \) is a tuple \( \langle a_i, v, t \rangle \) where agent \( a_i \) is prohibited from occupying vertex \( v \) at time step \( t \). A consistent path for agent \( a_i \) is a path that satisfies all of \( a_i \)'s constraints, and a consistent solution is a solution composed of only consistent paths. Note that a consistent solution can be invalid if despite the fact that the paths are consistent with the individual agent constraints, they still have inter-agent conflicts.

The high-level of CBS searches the constraint tree (CT). The CT is a binary tree, in which each node \( N \) contains: (1) A set of constraints imposed on the agents \( N.constraints \), (2) A single solution \( N.solution \) consistent with these constraints, (3) The cost of \( N.solution \) \( N.cost \).

The root of the CT contains an empty set of constraints. A successor of a node in the CT inherits the constraints of the parent and adds a single new constraint for a single agent. \( N.solution \) is found by the low-level search described below. A CT node \( N \) is a goal node when \( N.solution \) is valid, i.e., the set of paths for all agents have no conflicts. The high-level of CBS performs a best-first search on the CT where nodes are ordered by their costs \( N.cost \).

Processing a node in the CT: Given a CT node \( N \), the low-level search is invoked for individual agents to return an optimal path that is consistent with their individual constraints in \( N \). Any optimal single-agent path-finding algorithm can be used by the low level of CBS. We used A* with the true shortest distance heuristic (ignoring constraints). Once a consistent path has been found (by the low level) for each agent, these paths are validated with respect to the other agents by simulating the movement of the agents along their planned paths \( N.solution \). If all agents reach their goal without any conflict \( N \) is declared as the goal node, and \( N.solution \) is returned. If, however, while performing the validation, a conflict is found for two (or more) agents the validation halts and the node is declared as non-goal.
Resolving a conflict: the split action Given a non-goal CT node, \( N \), whose solution, \( N.\text{solution} \), includes a conflict, \( \langle a_i, a_j, v, t \rangle \), we know that in any valid solution at most one of the conflicting agents, \( a_i \) or \( a_j \), may occupy vertex \( v \) at time \( t \). Therefore, at least one of the constraints, \( (a_i, v, t) \) or \( (a_j, v, t) \), must be satisfied. Consequently, CBS splits \( N \) and generates two new CT nodes as children of \( N \), each adding one of these constraints to the previous set of constraints, \( N.\text{constraints} \). Note that for each (non-root) CT node the low-level search is activated for one agent only – the agent for which the new constraint was added.

CBS Example: Pseudo-code for CBS is shown in Algorithm 1. We cover it using the example in Figure 1(I), where the mice need to get to their respective pieces of cheese. The corresponding CT is shown in Figure 1(II). The root (\( R_1 \)) contains an empty set of constraints and the low-level search returns the following individual optimal paths: \( P_1 = \langle S_1, A, C, E, G_1 \rangle \) for agent \( a_1 \) and \( P_2 = \langle S_2, B, D, E, G_2 \rangle \) for agent \( a_2 \) (line 3). Thus, the total cost of \( R_1 \) is 8. \( R_1 \) is then inserted into the sorted OPEN-list and will be expanded next. When validating the two-agents solution (line 8), a conflict \( \langle a_1, a_2, E, 3 \rangle \) is found. As a result, \( R_1 \) is declared as non-goal. \( R_1 \) is split and two children are generated (via the \( \text{generate-child}() \) function, also shown in Algorithm 1) to resolve the conflict (line 15). The left child \( U \) adds the constraint \( \langle a_1, E, 3 \rangle \) while the right child \( V \) adds the constraint \( \langle a_2, E, 3 \rangle \). The low-level search is now invoked (line 20) for \( U \) to find an optimal path for agent \( a_1 \) that also satisfies the new constraint. For this, \( a_1 \) must wait one time step at \( C \) (or at \( S_1 \) or \( A \)) and the path \( \langle S_1, A, C, E, G_1 \rangle \) is returned for \( a_1 \). The path for \( a_2 \), \( \langle S_2, B, D, E, G_2 \rangle \) remains unchanged in \( U \). Since the cost of \( a_1 \) increased from 4 to 5 time steps the cost of \( U \) is now 9. As the sum of right child \( V \) is generated, also with cost 9. Both children are added to OPEN (line 16). In the final step \( U \) is chosen for expansion, and the underlying paths are validated. Since no conflicts exist, \( U \) is declared as a goal node (lines 8-10) and its solution is returned. Lines 12-13 are optional but speed up the search. They are the main contribution of this paper.

Meta-agent CBS (MA-CBS) MA-CBS(\( B \)) (Sharon et al. 2012b) is a generalization of CBS. When the number of conflicts between a given pair of agents exceeds a predefined parameter \( B \), then conflicting agents are merged into a meta-agent. Meta agents are treated as a joint composite agent by the low-level solver. Basic CBS is, in fact, MA-CBS(\( \infty \)), i.e., never merge agents.

Sensitivity of CBS Given a consistent \( h \)-function, \( A^* \) must expand all nodes with \( f < C^* \) (where \( C^* \) is the optimal solution cost) in order to guarantee optimality of the solution. No such mandatory nodes are known for CBS. In fact, CBS is very sensitive to the paths found by the low level and to the conflicts chosen to cause split as these can significantly influence the number of CT nodes. Consider the same example problem from Figure 1(I) but now assume that the paths found at the root of the CT by the low level are: \( P'_1 = \langle S_1, B, D, E, G_1 \rangle \) for \( a_1 \) and \( P'_2 = \langle S_2, B, D, E, G_2 \rangle \) for \( a_2 \) as shown in figure 1(III). If at the root (\( R_2 \)), the conflict \( \langle a_1, a_2, B, 1 \rangle \) is chosen to cause a split and the left child \( M \) is generated with path \( P''_1 = \langle S_1, A, C, E, G_1 \rangle \) (cost 8) but \( M \) includes another conflict \( \langle a_1, a_2, D, 2 \rangle \). It will also be split and its left child \( O \) with path \( P'_1 = \langle S_1, A, C, E, G_1 \rangle \), cost 8 is now identical to \( R_1 \) (the root node of figure 1(II)). In \( O \) the conflict \( \langle a_1, a_2, E, 3 \rangle \) must be chosen and this is done in the same manner as described for figure 1(II). Figure 1(II) (CT size 3) and figure 1(III) (CT size 7) correspond to running CBS on the same problem while choosing different paths and conflicts.

Improved CBS: Bypassing Conflicts When a conflict is found between the paths of two agents CBS immediately splits the corresponding CT node into two

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**Algorithm 1: High-level of CBS**

1. `Main(MAPF problem instance)`
2. \( R.\text{constraints} \leftarrow \emptyset \)
3. \( R.\text{solution} \leftarrow \text{find individual paths using low level} \)
4. \( R.\text{cost} \leftarrow \text{STC}(R.\text{solution}) \)
5. insert \( R \) to OPEN
6. while OPEN not empty do
7. \( N \leftarrow \text{best node from OPEN // lowest solution cost} \)
8. Validate the paths in \( N \) until a conflict occurs.
9. if \( N \) has no conflict then
10. \( \text{return } N.\text{solution // } N \text{ is goal} \)
11. \( C \leftarrow \text{first conflict } \langle a_i, a_j, v, t \rangle \text{ in } N \)
12. if \( \text{Find-bypass}(N, C) \) then
13. \( \text{Continue /Optional} \)
14. for each agent \( a_i \) in \( C \) do
15. \( A \leftarrow \text{generate child}(N, (a_i, s, t)) \)
16. \( \text{Insert } A \text{ to OPEN} \)
17. \( \text{Generate Child(Node } N, \text{ Constraint } C = \langle a_i, s, t \rangle) \)
18. \( A.\text{constraints} \leftarrow N.\text{constraints} + \langle a_i, s, t \rangle \)
19. \( A.\text{solution} \leftarrow N.\text{solution} \)
20. \( \text{Update } A.\text{solution} \text{ by invoking low level}(a_i) \)
21. \( A.\text{cost} \leftarrow \text{STC}(A.\text{solution}) \)
22. \( \text{return } A \)
children, each with its own new constraint. However, it is sometimes possible to prevent such a split and bypass the conflict by modifying the chosen path of one of the agents.

Consider again Figure 1(III) and assume that we start with the root of Figure 1(III) (R2 with paths P′ 1 and P2) where conflict (a1, a2, B, 1) is found. CBS splits R2 as described above and generates two successors. At the left child M path P′ 1 = (S1, A, D, E, G1) is assigned to agents a1. However, R2 can replace path P′ 1 for agent a1 with path P″ 1, as it satisfies all of R2.constraints and has the same cost. P″ 1 is a better path for a1 than path P′ 1 because the conflict at node B no longer occurs. Now, instead of a CT with 7 nodes we get a CT with only 5 nodes. Importantly, we note that since we aim to find the optimal solution, we only consider bypassing paths that have the same cost as the original path. Otherwise, to guarantee admissibility we must split the node.

Adding the option of bypassing requires only a small addition to the CBS pseudo code (lines 12-13 in Algorithm 1). After a conflict is found in node N, bypassing paths are searched for. If such a path is found, it is adopted by the corresponding CT node without the need to split N. Below, we provide a number of methods for finding such bypasses.

Definitions

1. For each CT node N we use N.NC to denote the total number of conflicts of the form (a1, a2, v, t) between the paths in N.solution. Calculating N.NC is trivial.

2. A path P′ i is a valid bypass to path P i for agent a i with respect to a conflict C = (a1, a2, v, t) and a CT node N, if the following conditions are satisfied: (i) Unlike P i, P′ i does not include conflict C, (ii) cost(P′ i) = cost(P i) and (iii) P i and P′ i are both consistent with N.constraints.

3. Replacing a path P i at a CT node N means replacing P i with a valid bypass P′ i for agent a i. When this happens we say that P i was adopted by N.

Adopting a valid bypass may introduce more conflicts compared to the original path and potentially lead to worse overall runtime. Thus, we only allow to adopt bypasses that reduce N.NC. These are called helpful bypasses. That is:

4. A valid bypass P′ i is a helpful bypass to P i if N′.NC < N.NC (where N′ is the CT node that adopted P′ i). We use the symbol “<” (and not “≤”) in definition 4, to avoid an infinite loop that alternates between conflicts.

Next we cover methods for finding adoptable bypasses.

Bypass1: Peek at the Child

Our first method, denoted as Bypass1 (BP1) peeks at either of the immediate children in the CT and tries to adopt their paths. This method was, in fact, demonstrated in our example of Figure 1(III) discussed above – where P′ 1 was adopted by the root. Once the left child (M) is generated we notice that path P″ 1 is a helpful bypass to path P′ 1 of R2. If P″ 1 is adopted by R2 it would avoid the need to split R2 and branch according to the conflict (a1, a2, B, 1). In this case node M and its sibling node N are not added to the CT.

Formally, BP1 works as follows. Let N be a CT node where the paths for agents a1 and a2 in N.solution are P i and P j, respectively. Assume that N.solution includes a conflict C = (a1, a2, v, t) that we want to bypass. We can now generate the left child and reveal the shortest path P′ i of a1 that satisfies the constraint (a1, v, t) by using the generate-child() function. If P′ i is a helpful bypass to P i wrt. conflict C, then P′ i is adopted by N without adding any new constraint to N. Importantly, while the left child of N was technically generated it will not be added to OPEN and to the CT. The right child will never be generated and the CT size is not increased. Similarly, if peeking at the left child failed because the resulting path was not a helpful bypass, the same peeking mechanism can be done for agent a2 at the right child. If both peek operations failed then we have these nodes at hand and insert them to OPEN normally.

While processing node N, BP1 doesn’t incur any extra overhead over basic CBS due to peek operations. Basic CBS generates both children and adds both to OPEN. In the worst case when both peek operations fail, BP1 does exactly the same as CBS, i.e., generates and adds these children to OPEN. But if one of these nodes was found helpful then BP1 doesn’t add new nodes to OPEN and might even avoid the need to generate and run a low-level search for the 2nd child.

The main reason for adopting a path from a child is that a split action is canceled and new nodes are not generated at this point. This can potentially save a significant amount of search due to a smaller size CT as shown in Figure 1.

Bypass2: Deep Search for Bypasses

Bypass2 (BP2) generalizes BP1. Pseudo code for both is provided in Algorithm 2. Define ST(N) as the subtree below N (including node N) containing only nodes with the same cost as N.cost. BP2 searches (in best-first manner according to N.NC) through the entire set of nodes in ST(N) (line 1) in order to find a helpful descendant, i.e., a descendant N′ ∈ ST(N) such that N′.NC < N.NC. In line 4 it calls the function generate-child (shown in Algorithm 1) which invokes the low-level. If the child is a helpful descendant (has the same cost but with fewer conflicts, line 5) then this low-level path is returned and adopted by N (lines 6-7).2 BP1 uses the same pseudo code except that in line 1 it only allows the two immediate children.

2This is called first-fit adoption. By contrast, best-fit adoption continues to search all nodes in ST(N) and the path of the helpful descendant with the smallest NC is returned. We can also parameterize the amount of search in ST(N) and halt once we reach a predefined threshold for the number of allowed nodes Q. BP1 is a special case of this where Q = 2, while BP2 is the case where
It is important to note that in practical implementation when BP2 failed to find a helpful descendant (line 9), then we have all the frontier nodes of this search at hand and they can be passed to Algorithm 1 which will directly add them to OPEN without the need to generate them again (and invoke the low-level again for these nodes).

It is also important to note that node N in its “new suit” after adopting the solution of one of its descendants will now be the best node in OPEN and will be chosen for expansion next. Therefore, if it again has a helpful descendant then the bypass process will be repeated here. In fact, the next real split will occur only when a bypass call fails.

**Experimental results**

**Performance of BP1 and BP2** We experimented with CBS and with MA-CBS(B) with various values for B. In general, in all these settings BP1 outperformed CBS by up to an order of magnitude. Nevertheless, in very rare cases (a few out of 25,000) slowdown was observed due to bad tie breaking and due to recurring conflicts. In some domains BP2 further outperformed BP1 but in others it was slightly worse.

Table 1 shows representative results (of MA-CBS(5)) for 100 random instances of a 5×5 grid with 15% obstacles. As was done by Sharon et al. (2013; 2012a), we report the success rate - the number of instances solved by each of the algorithms within 5 minutes. We also report the average runtime (in ms) over instances solved by all three algorithms within 5 minutes (shown in the Ins column). BP1 clearly outperformed CBS in its success rate and provided speedup of up to a factor of 5 (for 9 agents). BP2 incurs more overhead than BP1 in its search but could not find more bypasses as this domain is dense with agents and most bypasses were found by BP1. Thus, it was generally worse than BP1.

Figure 2 shows the success rate over 300 instances from the three standard benchmark maps (brc202d, den520d, ost003d) of the game Dragon Age: Origins (DAO) (Sturtevant 2012) that were used by Sharon et al. (2013; 2012a), 100 instances per map. It also shows the average CPU time (in seconds) for all instances that were solved by all three algorithms up to 55 agents and for the BP variants only, for 60 agents or more; hence the jump in the curves at 60. Here too, BP1 significantly outperformed CBS by up to an order of magnitude (e.g., 50 agents). This domain is less dense with agents and many times BP1 could not find bypasses.

**Table 1:** 5×5 grid, MA-CBS(5)

<table>
<thead>
<tr>
<th>k</th>
<th>Ins.</th>
<th>Runtime (ms)</th>
<th>Success rate %</th>
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<tbody>
<tr>
<td></td>
<td></td>
<td>CBS</td>
<td>BP1</td>
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<tr>
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</table>

When CBS is used, adding BP on top is a great enhancement. This was demonstrated empirically on standard benchmarks. Future work will (1) further investigate search for bypasses (2) try to find better low-level paths in the first place (3) compare and better understand the pros and cons of the various MAPF algorithms under different circumstances.

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**Conclusions and Future Work**

When CBS is used, adding BP on top is a great enhancement. This was demonstrated empirically on standard benchmarks. Future work will (1) further investigate search for bypasses (2) try to find better low-level paths in the first place (3) compare and better understand the pros and cons of the various MAPF algorithms under different circumstances.
References


